

# Scattering of Scalar Waves by Schwarzschild Black Hole Immersed in Magnetic Field

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The magnetic field is one of the most important constituents of the cosmic space and one of the main sources of the dynamics of interacting matter in the universe. The astronomical observations imply the existence of a strong magnetic fields of up to  $10^4 - 10^8 G$  near supermassive black holes in the active galactic nuclei and even around stellar mass black holes. In this paper, with the quantum scattering theory, we analysis the Schrödinger-type scalar wave equation of black hole immersed in magnetic field and numerically investigate its absorption cross section and scattering cross section. We find that the absorption cross sections oscillate about the geometric optical value in the high frequency regime. Furthermore in low frequency regime, the magnetic field makes the absorption cross section weaker and this effect is more obviously on lower frequency brand. On the other hand, for the effects of scattering cross sections for the black hole immersed in magnetic field, we find that the magnetic field makes the scattering flux weaker and its width narrower in the forward direction. We find that there also exists the glory phenomenon along the backforward direction. At fixed frequency, the glory peak is higher and the glory width becomes narrower due to the black hole immersed in magnetic field.

**Keywords:** absorption cross section, scattering cross section, magnetic field.

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## I. INTRODUCTION

It is well known that general relativity and quantum mechanics are incompatible in their current form. However, after Hawking found that black holes can emit, as well as scatter, absorb, and that the evaporation rate is proportional to the total absorption cross section. A lot of scholars are interest in the absorption of quantum fields by black hole since 1970s. By using numerical methods, Sanchez [1, 2] found that the absorption cross section of massless scalar wave exhibits oscillation around the geometry-optical limit characteristic of diffraction patterns by Schwarzschild black hole. Unruh [3] showed that the scattering cross section for the fermion is exactly  $1/8$  of that for the scalar wave in the low-energy limit. By numerically solving the single-particle Dirac equation in Painlevé-Gullstrand coordinates, Chris Doran et al [4] studied the absorption of massive spin-half particle by a small Schwarzschild black hole and they found oscillations around the classical limit whose precise form depends on the particle mass. Crispino et al [5] have computed numerically the absorption cross section of electromagnetic waves for arbitrary frequencies and have found that its high-frequency behavior is very similar to that for massless scalar field by Schwarzschild black hole. In last several years, Oliveria et al [6] extended to study the absorption of planar in a draining bathtub, the absorption cross section of sound waves with arbitrary frequencies in the canonical acoustic hole spacetime [7] and electromagnetic absorption cross section from Reissner-Nordström black holes [8]. Recently, absorption cross section (or

gray body factors) has been of interest in the context of higher-dimensional using standard field theory in curved spacetimes [9–11] and effective string model [12].

The magnetic field is one of the most important constituents of the cosmic space and one of the main sources of the dynamics of interacting matter in the universe. In addition some other theories [13–15] imply the existence of a strong magnetic fields of up to  $10^4 - 10^8 G$  near supermassive black holes in the active galactic nuclei and even around stellar mass black holes. In order to make estimations of possible influence of the magnetic field on the supermassive black holes, we need the two parameters at hand: the magnetic field parameter  $B$  and the mass of the black hole  $M$ . Interaction of a black hole and a magnetic field can happen in a lot of physical situations: when an accretion disk or other matter distribution around black hole is charged; when taking into consideration galactic and intergalactic magnetic fields, and, possibly, if mini-black holes are created in particle collisions in the brane-world scenarios. So astrophysics have highly interest to investigate the magnetic fields around black holes [16]. A magnetic field is important as a background field testing black hole geometry. A magnetic field near a black hole leads to a number of processes, such as extraction of rotational energy from a black hole, known as the Blandford-Znajek effect [17], negative absorption (masers) of electrons [18]. At the classical level, the magnetic perturbation can also be described by its damped characteristic modes, which called the quasinormal modes (QNMs) [19–22] which could be observed in experiments, and by the scattering properties, which are encoded in the S-matrix of the perturbation. All of these effects are usually called the "fingerprints" of a black hole. In recent few years, we all know that quasinormal modes of black holes has gained considerable attention

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because of their applications in string theory through the AdS/CFT correspondence.

In this paper we mainly focus on the scalar scattering process of black hole immersed in magnetic field and how the interaction of black hole and strong magnetic field effects on scalar absorption and scattering cross sections. The outline of this paper is as follows: In Sec.II, we set up scalar field equation black holes immersed in a magnetic field and analysis effective potential. In the Sec. III and IV, we concentrate on the absorption and scattering cross section of the scalar wave by black holes immersed in a magnetic field. In the last section, a brief conclusion is given.

## II. SCALAR FIELD EQUATION AND EFFECTIVE POTENTIAL

The Diaz and Ernst solution [23] describing the black holes immersed in a magnetic field takes the follow form:

$$ds^2 = \Lambda^2 \left[ \left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 - r^2 d\theta^2 \right] - \frac{r^2 \sin^2 \theta}{\Lambda^2} d\phi^2, \quad (1)$$

where the external magnetic field is determined by the parameter  $B$

$$\Lambda = 1 + \frac{1}{4} B^2 r^2 \sin^2 \theta, \quad (2)$$

and the unit magnetic field measured in  $Gs$  is  $B_M = 1/M = 2.4 \times 10^{19} \frac{M_{Sun}}{M}$ .

The general perturbation equation for the massless scalar field  $\Psi$  in the curve spacetime is given by

$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu) \Psi = 0. \quad (3)$$

For very strong magnetic fields in centres of galaxies or in colliders, corresponds to  $M \ll M$  in our units, so that one can safely neglect terms higher than  $B^2$  in Eq.(3). Indeed, in the expansion of  $\Lambda^4$  in powers of  $B$ , the next term after that proportional to  $B^2 r^2$ , is  $\sim B^4 r^4$  and, thereby, is very small in the region near the black hole. The term  $B^4 r^4$  is growing far from black hole, and, moreover the potential in the asymptotically far region is diverging, what creates a kind of confining by the magnetic field of the Ernst solution. This happens because the non-decaying magnetic field is assumed to exist everywhere in the universe. Therefore it is clear that in order to estimate a real astrophysical situation, one needs to match the Ernst solution with a Schwarzschild solution at some large  $r$ . Fortunately we do not need to do this for the scattering problem: the scattering properties of astrophysical interest is stipulated by the behavior of the effective potential in some region near black hole, while its behavior far from black hole is insignificant [24]. In

this way we take into consideration only dominant correction due-to magnetic field to the effective potential of the Schwarzschild black hole. By neglecting terms  $B^4$  and higher order terms and separating the angular variables, we reduce the wave equation (3) to the Schrödinger wave equation. The Klein-Gordon equation can be written in the spacetime (1) as

$$\frac{1}{1 - \frac{2M}{r}} \frac{\partial^2 \Psi}{\partial t^2} - \frac{1}{r^2} \frac{\partial}{\partial r} \left[ \left(1 - \frac{2M}{r}\right) r^2 \frac{\partial \Psi}{\partial r} \right] + \frac{1}{r^2} \nabla^2 \Psi = 0. \quad (4)$$

The positive-frequency solutions of Eq.(4) take as follows

$$\Psi_{\omega lm} = [\psi_{\omega l}(r)/r] Y_{lm} e^{-i\omega t}, \quad (5)$$

where  $Y_{lm}$  are scalar spherical harmonic functions and  $l$  and  $m$  are the corresponding angular momentum quantum numbers. In this case, the functions  $\psi_{lm}(r)$  satisfy the follow differential equation

$$\left(1 - \frac{2M}{r}\right) \frac{d}{dr} \left[ \left(1 - \frac{2M}{r}\right) \frac{d\psi_{\omega l}}{dr} \right] + [\omega^2 - V_{eff}^{(l)}(r)] \psi_{\omega l} = 0, \quad (6)$$

where

$$V_{eff}^{(l)}(r) = \left(1 - \frac{2M}{r}\right) \left[ \frac{l(l+1)}{r^2} + \frac{2M}{r^3} + 4B^2 m^2 \right]. \quad (7)$$

The effective potential  $V_{eff}^{(l)}(r)$  is plotted in Fig.1 for  $l = 0, 1, 2$ . From this figure, we can see that the effective potential  $V_{eff}^{(l)}(r)$  depends only on the values of  $r$ , angular quantum number  $l$ , ADM mass  $M$ , magnetic field  $B$ , respectively, and that the peak value of potential barrier gets upper and the location of the peak point ( $r = r_p$ ) moves along the right when the angular momentum  $l$  increases. We can find that the the height of the effective scattering potential increases as the angular momentum  $l$  increases. If we introduce the tortoise coordinate

$$x = \int \left(1 - \frac{2M}{r}\right)^{-1} dr, \quad (8)$$

The effective potentials  $V_{eff}^{(l)}(r)$  are changed into  $V_{eff}^{(l)}(x)$ , which are showed in Fig.2 for  $l = 0, 1, 2$ , it's obvious that they act as the typical scattering barriers in quantum mechanics theory. We see that the peak value of potential barrier gets upper and the location of the peak point ( $x = x_p$ ) moves along the right when the angular momentum  $l$  increases. We also find that the height of the effective scattering barrier increases as the magnetic field  $B$  increases, at the same time we can see that the height of the effective scattering barrier, affecting by the magnetic field, becomes higher than that of Schwarzschild black hole.

After introducing this coordinate transition, we can obtain the following Schrödinger-type equation

$$\frac{d^2 \psi_{\omega l}}{dx^2} + [\omega^2 - V_{eff}^{(l)}(x)] \psi_{\omega l} = 0. \quad (9)$$

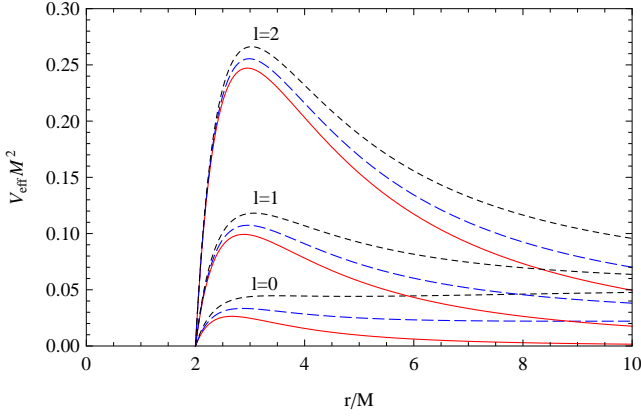


FIG. 1: (color online). The effective scattering potential  $V_{eff}(r)$  given by Eq.7 for scalar waves by the black hole immersed in magnetic field with  $l = 0, 1, 2$  for  $B = 0$  (red solid line, i.e. Schwarzschild case) and  $B = 0.08$  (blue dashed line), and  $B = 0.12$  (black dotted line).

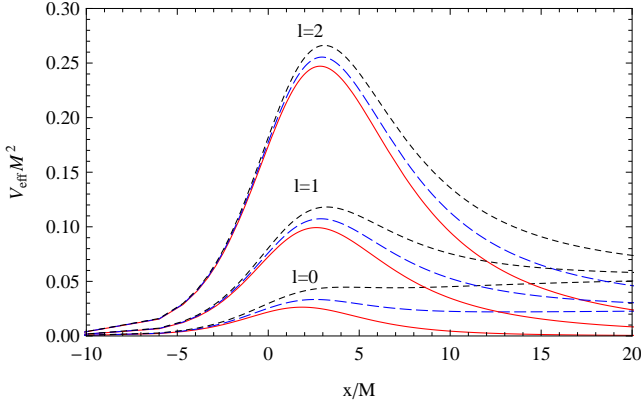


FIG. 2: (color online). The effective scattering potential  $V_{eff}(x)$  given by Eq.7 for scalar waves by the black hole immersed in magnetic field in tortoise coordinate with  $l = 0, 1, 2$  for  $B = 0$  (red solid line, i.e. Schwarzschild case) and  $B = 0.08$  (blue dashed line), and  $B = 0.12$  (black dotted line). From this figure, we can see the effective scattering potential  $V_{eff}(x)$  act as the typical scattering barrier in quantum mechanics theory.

The perturbation must be purely ingoing at the black hole event horizon  $r = r_+$ . So while  $r \rightarrow r_+$  i.e.  $x \rightarrow -\infty$ , we impose the boundary condition

$$\psi_{\omega l} = A_{\omega l}^{tr} e^{-i\omega x}, \text{ for } x \rightarrow -\infty. \quad (10)$$

It is straightforward to check that in the original coordinate system (1) the ingoing solution  $e^{-i\omega x}$  is well defined at  $r = r_+$ , whereas the out going solution  $e^{+i\omega x}$  is divergent. Towards spatial infinity, the asymptotic form of the solution is

$$\psi_{\omega l} = \omega x [A_{\omega l}^{in} (-i)^{l+1} h_l^{(1)*}(\omega x) + A_{\omega l}^{out} (i)^{l+1} h_l^{(1)}(\omega x)] \text{ for } x \rightarrow +\infty, \quad (11)$$

where  $h_l^{(1)}(\omega x)$  are spherical Bessel functions of the third kind [25], at the same time  $A_{in}$  and  $A_{out}$  are complex constants. We note that  $h_l^{(1)}(\omega x) \approx (-i)^{l+1} e^{ix}/x$  as  $x \rightarrow \infty$  and that the effective potential goes to zero as  $x \rightarrow -\infty$ , so we obtain

$$\psi_{\omega l} \approx \begin{cases} A_{\omega l}^{tr} e^{-i\omega x}, & \text{for } x \rightarrow -\infty; \\ A_{\omega l}^{in} e^{-i\omega x} + A_{\omega l}^{out} e^{+i\omega x}, & \text{for } x \rightarrow +\infty. \end{cases} \quad (12)$$

with the conserved relation

$$|A_{\omega l}^{tr}|^2 + |A_{\omega l}^{out}|^2 = |A_{\omega l}^{in}|^2 \quad (13)$$

The phase shift  $\delta_l$  is defined by

$$e^{2i\delta_l} = (-1)^{l+1} A_{out}/A_{in}. \quad (14)$$

In order to investigate the absorption cross section and scattering cross section, we must numerically solve the radial equation (9) under the boundary conditions Eq.(10) and Eq.(11), then compute the ingoing and outgoing coefficients  $A_{\omega l}^{in}$  and  $A_{\omega l}^{out}$  by matching onto Eq.(14) to give out the numerical phase shift.

### III. ABSORPTION CROSS SECTION

Base on the quantum mechanics theory, we know that the total absorption cross section is

$$\sigma_{abs} = \frac{\pi}{\omega^2} \sum_{l=0}^{\infty} (2l+1) (1 - |e^{2i\delta_l}|^2), \quad (15)$$

so we can define the partial absorption cross section as

$$\sigma_{abs}^{(l)} = \frac{\pi}{\omega^2} (2l+1) (1 - |e^{2i\delta_l}|^2), \quad (16)$$

and the absorption cross section have relation

$$\sigma_{abs}(\omega) = \sum_{l=0}^{\infty} \sigma_{abs}^{(l)}(\omega) = \frac{\pi}{\omega^2} \sum_{l=0}^{\infty} (2l+1) |T_{\omega l}|^2. \quad (17)$$

By using *mathematica* program, we straightforwardly compute values of ingoing and outgoing coefficients  $A_{\omega l}^{in}$  and  $A_{\omega l}^{out}$ . Then from Eq.(14), Eq.(15) and Eq.(16), we can simulate the partial absorption cross sections and their total absorption cross sections of the scalar field from the black hole immersed in magnetic field.

In Fig.3 we show the partial absorption cross sections  $\sigma_{abs}^{(l)}$ , i.e.  $l = 0, 1, 2$ , by the black hole immersed in magnetic field for different magnetic parameters  $B = 0.1$  and  $\Lambda = 0, 0.08$  and  $0.12$ . We find that the S-wave ( $l = 0$ ) contribution is responsible for the nonvanishing cross section in the zero-energy limit. Furthermore, by comparing different  $l$  partial absorption cross section curves, we find that the larger the value of  $l$  is, the smaller the corresponding value of  $\sigma_{abs}^{(l)}$  is. This is compatible with the fact that the scattering barrier  $V_{eff}$  is bigger

or larger values of  $l$ , which is showed in Fig.1 and Fig.2. These properties are similar to other black hole scattering system[2, 7, 26]. On the other hand with phase-integral method, Andersson [27] had gotten very similar results (see Fig.7 therein).

In order to consider effects of magnetic field on the partial absorption cross section. In Fig.4 we plot the partial absorption cross section for  $l = 0, 1, 2$  with  $B = 0$  ( i.e. Schwarzschild black hole case),  $B = 0.08$  and  $B = 0.12$ . We see that the magnetic field make the absorption weaker, even for low frequency mode. This is agree with the fact that the magnetic field is stronger, the higher value of the effective scattering barrier peak is for a fixed value of  $l$ , which can be seen in Fig.1 and Fig.2. But for high enough values of the frequency, the magnetic field does not effect the partial absorption cross section obviously. From Eq.(17), we know the absorption cross section have relation with the transmission coefficients [28]. This feature can be tested the transmission coefficients in Fig.5, where we find that high enough values of the frequency all transmission coefficients with fixed  $l$  tend to the unity. These properties help us understand the absorption process better.

In Fig.6 we plot total absorption cross sections  $\sigma_{abs}$  which contribute from  $l = 0$  to  $l = 5$  by the black hole immersed in magnetic field with fixed parameters  $B = 0$  ( i.e. Schwarzschild case),  $B = 0.08$  and  $B = 0.12$ . We can see that I) between the intermediate regime  $\omega M \sim (0.4, 1)$ , the contributions from the partial absorption sections create a regular oscillatory pattern. Each maximum in the oscillation of the total absorption cross section is linked to the maximum of a particular partial wave. II) If the wavelength of the incoming wave is much smaller than the black hole horizon (i.e.  $\omega M \gg 1$ ), the absorption cross section tends to the geometry-optical limit of  $\sigma_{abs}^{hf} = \pi b_c^2$ . This is verified by the total absorption cross section for the massless scalar field which was computed by Sanchez [2] in last century. At the same time, these properties are also found for electromagnetic wave absorption cross section [5] and for Fermion absorption cross section in the Schwarzschild black hole [4].

In bottom-left position of Fig.6, we plot total absorption cross sections for different values of magnetic parameters. We also consider the contributions of the angular momentum from  $l = 0$  to  $l = 5$  in Eq.(16). We can see that big values of the magnetic parameter  $B$  correspond to low total absorption cross section which is consistent with the fact of the partial section in Fig.4. and the scattering barrier which is showed in Fig.1 and 2. But we can find that the absorption cross sections oscillate about the geometric optical value in the high frequency regime. However in low frequency regime, the magnetic field makes the absorption cross section weaker, i.e. the magnetic makes obvious effect on lower frequency brand, not on high frequency brand. We note that this is a general result for massless scalar waves in Reissner-Nordström black hole [26] and for the minimally-coupled massless scalar wave in stationary black hole spacetimes

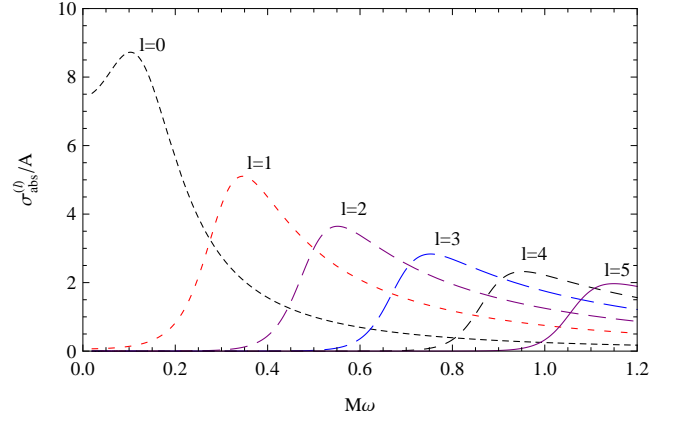


FIG. 3: (color online). The behavior of the partial absorption cross section  $\sigma_{abs}^{(l)}$ , from  $l = 0$  to  $l = 5$  for scalar waves by the black hole immersed in magnetic field.

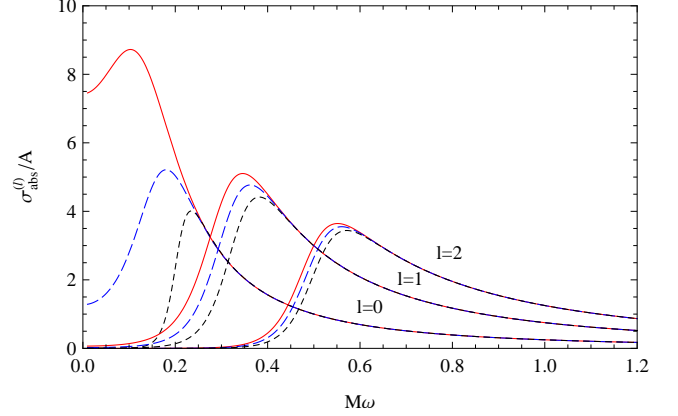


FIG. 4: (color online). The behavior of the partial absorption cross section  $\sigma_{abs}^{(l)}$ , from  $l = 0, 1, 2$  for scalar waves by the black hole immersed in magnetic field with  $B = 0$  (red solid line, i.e. Schwarzschild black hole case) and  $B = 0.08$  (blue dashed line), and  $B = 0.12$  (black dotted line).

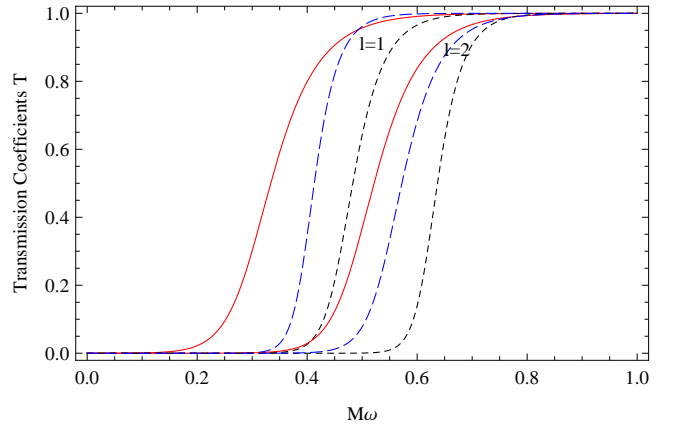


FIG. 5: (color online). The transmission coefficients with  $l = 1, l = 2$  are showed for different magnetic field with  $B = 0$  (red solid line, i.e. Schwarzschild black hole case) and  $B = 0.08$  (blue dashed line), and  $B = 0.12$  (black dotted line).

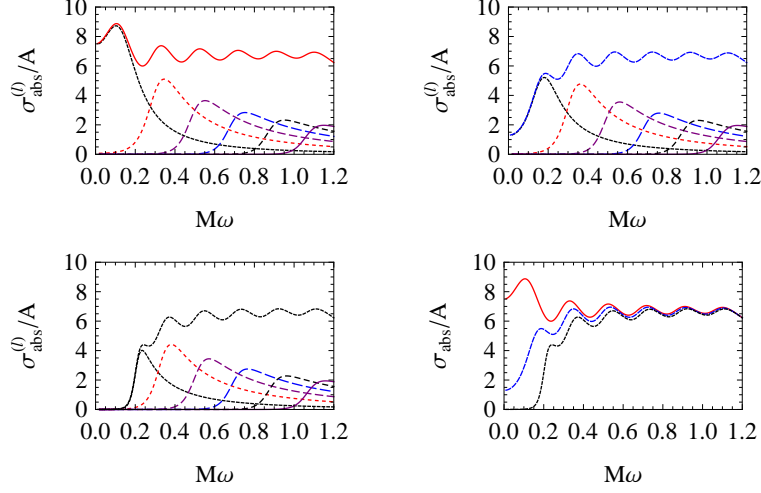


FIG. 6: (color online). The behavior of the partial absorption cross section  $\sigma_{abs}^{(l)}$  and  $\sigma_{abs}^{total}$  by the black hole immersed in magnetic field with  $B = 0$  (top-left i.e. Schwarzschild black hole case),  $B = 0.08$  (top-right),  $B = 0.12$  (bottom-left), and their corresponding total absorption cross sections  $\sigma_{abs}^{total}$  (bottom-right).

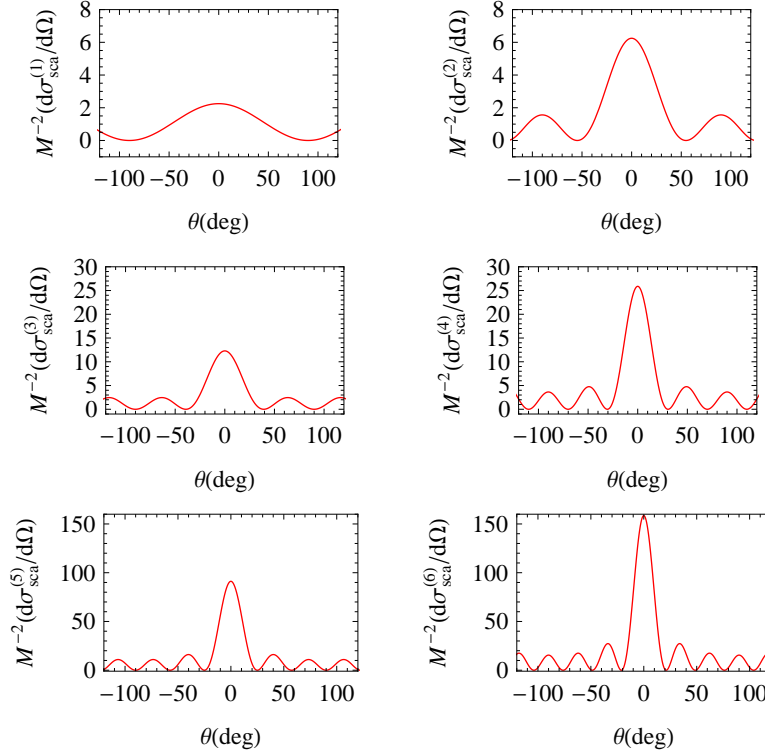


FIG. 7: (color online). The behavior of the partial scattering cross sections  $\sigma_{sca}^{(l)}$ , from  $l = 1$  to  $l = 6$ , at  $M\omega = 1$  for the scalar wave is scattered by the black hole immersed in magnetic field with  $B = 0.2$ .

[29]. There are similar properties for total absorption section from the charged black hole coupling to Born-Infeld electrodynamics [30] and dark energy [31] .

#### IV. SCATTERING CROSS SECTION

From the quantum mechanics theory, it's well known that the scattering amplitude is expressed as

$$f(\theta) = \frac{1}{2i\omega} \sum_{l=0}^{\infty} (2l+1) [e^{2i\delta_l} - 1] P_l(\cos\theta). \quad (18)$$

From this scattering amplitude, we can give the differential scattering cross section immediately

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2. \quad (19)$$

At last we can define the scattering and absorption cross sections [32, 33]

$$\sigma_{sca} = \int \frac{d\sigma}{d\Omega} d\Omega = \frac{\pi}{\omega^2} \sum_{l=0}^{\infty} (2l+1) |e^{2i\delta_l} - 1|^2, \quad (20)$$

so the partial scattering cross section is

$$\sigma_{sca}^{(l)} = \frac{\pi}{\omega^2} (2l+1) |e^{2i\delta_l} - 1|^2. \quad (21)$$

In order to simulate the scattering cross sections (20)-(21), we must numerically solve differential equation (9) under boundary conditions (10) and (11), to obtain numerical values for the phase shifts via Eq.(14).

Figures 7 show the partial scattering cross section a function of angle for six different partial waves from  $l = 1$  to  $l = 6$ . By comparing these figures, we can see that, when the  $L$  increases, the flux is preferentially scattered in the forward direction, i.e. the scattering angle width become narrower. At the time a more complicated pattern arises and we find a damping oscillation pattern. The similar properties are observed for black hole scattering [33–35]. The explanation for the physical origin of the oscillations can be found in Ref.[36].

Figures 8, 9 and 10 compare the scattering cross sections for the Schwarzschild black hole with the black hole immersed in magnetic field with  $B = 0.2$  and  $0.3$ . We find that the magnetic field makes the scattering flux weaker and its width narrower in the forward direction. In the other words, the scalar field scattering becomes more diffusing due to the black hole immersed in magnetic field. In Fig.10 we can see that there exists the glory phenomenon along the backforward direction [26, 33]. At fixed frequency, the glory peak is higher and the glory width becomes narrower due to the black hole immersed in magnetic field. So we can find that even the scalar field scattering becomes more diffusing due to the black hole immersed in magnetic field, but the glory phenomenon along the backforward direction becomes better for astronomy observation.

## V. CONCLUSIONS

In this paper we have investigated the scattering and absorption cross section of the scalar wave by the black hole immersed in magnetic field. We found that the magnetic parameter  $B$  makes the absorption cross section lower which is consistent with the fact of the scattering barrier which is showed in Fig.1 and 2. We also found that the absorption cross sections oscillate about the geometric optical value in the high frequency regime. However in low frequency regime, the magnetic field makes

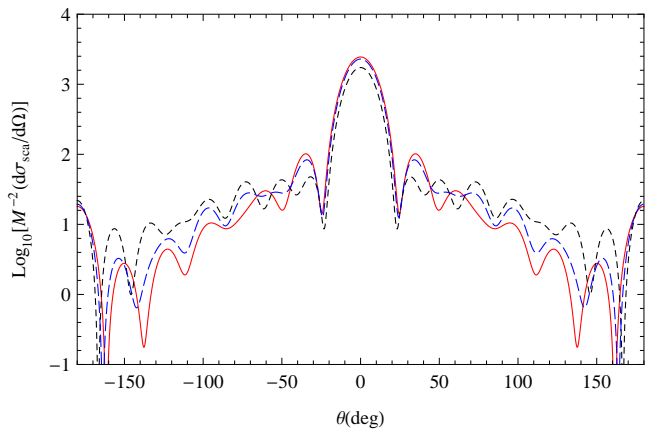


FIG. 8: (color online). The behavior of the total scattering cross sections  $\sigma_{sca}^{(l)}$  at  $M\omega = 1$  between  $(-180^\circ-180^\circ)$  for the scalar wave is scattered by the black hole immersed in magnetic field with  $B = 0$  (red solid line, i.e. Schwarzschild case) and  $B = 0.2$  (blue dashed line), and  $B = 0.3$  (black dotted line).

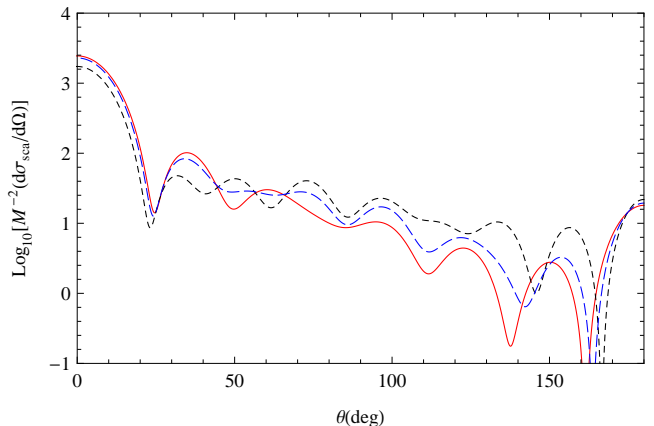


FIG. 9: (color online). The behavior of the total scattering cross sections  $\sigma_{sca}^{(l)}$  at  $M\omega = 1$  between  $(0^\circ-180^\circ)$  for the scalar wave is scattered by the black hole immersed in magnetic field with  $B = 0$  (red solid line, i.e. Schwarzschild case) and  $B = 0.2$  (blue dashed line), and  $B = 0.3$  (black dotted line).

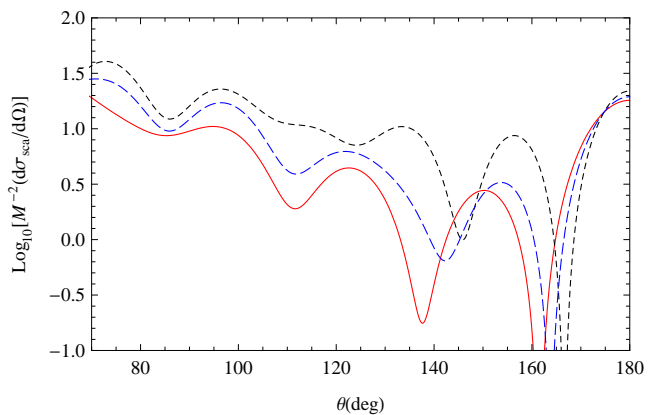


FIG. 10: (color online). The behavior of the total scattering cross sections  $\sigma_{sca}^{(l)}$  at  $M\omega = 1$  between  $(60^\circ-180^\circ)$  for the scalar wave is scattered by the black hole immersed in magnetic field with  $B = 0$  (red solid line, i.e. Schwarzschild case) and  $B = 0.08$  (blue dashed line), and  $B = 0.12$  (black dotted line).

the absorption cross section weaker and this effect is more obviously on lower frequency band. For the effects of the scattering cross sections for the black hole immersed in magnetic field, we found that the magnetic field makes the scattering flux weaker and its scattering width narrower in the forward direction. At the same time we found that there also exists the glory phenomenon along the backforward direction. At fixed frequency, the glory peak is higher and the glory width becomes narrower due to the black hole immersed in magnetic field. So the glory phenomenon along the backforward direction becomes better for astronomy observation.

Just as the Brazil physicist Crispino et al [26] have pointed out: "In principle, highly accurate measurements of, for example, the gravitational wave flux scattered by a black hole could one day be used to estimate the black holes charge. A more immediate possibility is

that scattering and absorption patterns may be observed with black hole analog systems created in the laboratory. Even if experimental verification is not forthcoming, we hope that studies of wave scattering by black holes will continue to improve our understanding of how black holes interact with their environments."

## VI. ACKNOWLEDGMENTS

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